PERK: Compact Signature Scheme Based on a New Variant of the Permuted Kernel Problem

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Permuted Kernel Problem

Definition (IPKP [Sha90])

Let $m < n$ be positive integers, Given

- $\boldsymbol{H} \in \mathbb{F}_q^{m \times n}$;
- $\boldsymbol{x} \in \mathbb{F}_q^{n}$;
- $\boldsymbol{y} \in \mathbb{F}_q^{m}$,

the Inhomogeneous Permuted Kernel Problem $\mathrm{IPKP}_{q,m,n}$ asks to find a permutation $\pi \in \mathcal{S}_n$ such that

$$\boldsymbol{H}\pi[\boldsymbol{x}] = \boldsymbol{y}.$$

A variant of the Permuted Kernel Problem

Definition (r-IPKP)

Let $m < n$ and t be positive integers, Given

- $\boldsymbol{H} \in \mathbb{F}_q^{m \times n}$;
- $(\boldsymbol{x}_1, \ldots, \boldsymbol{x}_t) \in (\mathbb{F}_q^n)^t$;
- $(\boldsymbol{y}_1, \ldots, \boldsymbol{y}_t) \in (\mathbb{F}_q^m)^t$,

the Relaxed Inhomogeneous Permuted Kernel Problem r-IPKP$_{q,m,n,t}$ asks to find a permutation $\pi \in \mathcal{S}_n$ such that

$$\boldsymbol{H}\pi\Big[\sum_{i\in[1,t]} \kappa_i \boldsymbol{x}_i\Big] = \sum_{i\in[1,t]} \kappa_i \boldsymbol{y}_i$$

for some $(\kappa_1, \ldots, \kappa_t) \in (\mathbb{F}_q)^t \setminus \{(0, \ldots, 0)\}$.

Multi-dimensional IPKP

Definition (IPKP [LP11])

Let $m < n$ and t be positive integers, Given

- $\boldsymbol{H} \in \mathbb{F}_q^{m \times n}$;
- $(\boldsymbol{x}_1, \ldots, \boldsymbol{x}_t) \in (\mathbb{F}_q^n)^t$;
- $(\boldsymbol{y}_1, \ldots, \boldsymbol{y}_t) \in (\mathbb{F}_q^m)^t$,

the Inhomogeneous Permuted Kernel Problem $\mathrm{IPKP}_{q,m,n,t}$ asks to find a permutation $\pi \in \mathcal{S}_n$ such that

$$\boldsymbol{H}\pi[\boldsymbol{x}_i] = \boldsymbol{y}_i$$

for all $i \in [1, t]$.

Outline

Outline

MPC-in-the-Head

Generic paradigm by Ishai, Kushilevitz, Ostrovsky, and Sahai [IKOS07, IKOS09].

$$\text{MPC protocol} \implies \text{ZK-proof}$$

1. Prover splits secret and commits to the states;
2. Verifier sends a random challenge γ;
3. Prover simulates locally ("in the head") all the parties, and commits to the views;
4. Verifier chooses a random party i^* and asks to reveal all the views except i^*;
5. Verifier finally checks the views are consistent and with an honest execution of the MPC protocol.

MPC-in-the-Head and PKP

Name	Type	σ size
SUSHYFISH [Beu20]	5-round with helper	$\sim$12 kB
[BG22]	5-round using structure	$\sim$9 kB
[Fen22]	7-round	$\sim$13 kB

Table: Comparison of recent digital signature schemes based on PKP assumptions

Parameters in [BG22]

- PKP parameters $(q, n, m) \implies$ attacks on IPKP
- MPC parameters $(N, \tau) \implies$ KZ attack on 5-round protocols [KZ20]

KZ attack cost depends on the challenge space (the number of possibilities for γ).
Increasing the challenge space leads to a decrease in τ.

	[BG22]	our work
Challenge space	$\mathbb{F}_q$	$\mathbb{F}_q^t$

Our parameters

Parameter Set	PKP parameters					MPC param.		pk size	σ size
	λ	q	n	m	t	N	τ		
[BG22]-fast	128	997	61	38	1	32	42	0.15 kB	9.90 kB
[BG22]-short	128	997	61	38	1	256	31	0.24 kB	8.81 kB
PERK-I-fast3	128	1021	79	35	3	32	30	0.15 kB	8.35 kB
PERK-I-fast5	128	1021	83	36	5	32	28	0.24 kB	8.03 kB
PERK-I-short3	128	1021	79	35	3	256	20	0.15 kB	6.56 kB
PERK-I-short5	128	1021	83	36	5	256	18	**0.24 kB**	**6.06 kB**
PERK-III-fast3	192	1021	112	54	3	32	46	0.23 kB	18.8 kB
PERK-III-fast5	192	1021	116	55	5	32	43	0.37 kB	18.0 kB
PERK-III-short3	192	1021	112	54	3	256	31	0.23 kB	15.0 kB
PERK-III-short5	192	1021	116	55	5	256	28	0.37 kB	13.8 kB
PERK-V-fast3	256	1021	146	75	3	32	61	0.31 kB	33.3 kB
PERK-V-fast5	256	1021	150	76	5	32	57	0.51 kB	31.7 kB
PERK-V-short3	256	1021	146	75	3	256	41	0.31 kB	26.4 kB
PERK-V-short5	256	1021	150	76	5	256	37	0.51 kB	24.2 kB

Table: Parameters of PERK signature scheme

Performances

Parameter Set	Keygen	Sign	Verify
PERK-I-fast3	77 k	7.6 M	5.3 M
PERK-I-fast5	88 k	7.2 M	5.1 M
PERK-I-short3	80 k	39 M	27 M
PERK-I-short5	92 k	36 M	25 M
PERK-III-fast3	167 k	16 M	13 M
PERK-III-fast5	184 k	15 M	12 M
PERK-III-short3	174 k	82 M	65 M
PERK-III-short5	194 k	77 M	60 M
PERK-V-fast3	297 k	36 M	28 M
PERK-V-fast5	322 k	34 M	27 M
PERK-V-short3	299 k	184 M	142 M
PERK-V-short5	329 k	170 M	131 M

Table: Performances of our implementation for different instances of PERK. The key generation numbers are in kilo CPU cycles, while the signing and verification numbers are in million CPU cycles.

Outline

Number of solutions

Proposition

The average number of solutions for a random $\mathrm{IPKP}_{q,m,n}$ *instance is*

$$\frac{n!}{q^m}$$

Since all existing attacks on IPKP and variants are combinatorial, they benefit from a speedup equal to $\max(1, \frac{n!}{q^m})$.

$\Longrightarrow$ equivalent to Gilbert-Varshamov bound

Georgiades algorithm [Geo92]

$$\begin{pmatrix} I_m & H' \end{pmatrix} \begin{pmatrix} x_1 \\ \hline x_2 \end{pmatrix} = y$$

$$x_1 = y - H' x_2$$

$\Rightarrow$ enumerate x_2 as every subpermutation of x of size $n - m$.

Georgiades algorithm [Geo92]

Proposition (Complexity)

$$\mathcal{T} = \mathcal{O}\left(\frac{n!}{(n-m)!}\right)$$

$\Longrightarrow$ equivalent to Prange

Time-memory trade-off

$$\begin{pmatrix} \boldsymbol{H}_1 & \boldsymbol{H}_2 \end{pmatrix} \begin{pmatrix} \boldsymbol{x}_1 \\ \hline \boldsymbol{x}_2 \end{pmatrix} = \boldsymbol{y}$$

$$L_1 = \{(\boldsymbol{x}_1, \boldsymbol{H}_1\boldsymbol{x}_1) \,|\, \boldsymbol{x}_1 \in \mathbb{F}_q^{n/2} \text{ sub-permutation of } \boldsymbol{x}\}$$

$$L_2 = \{(\boldsymbol{x}_2, \boldsymbol{y} - \boldsymbol{H}_2\boldsymbol{x}_2) \,|\, \boldsymbol{x}_2 \in \mathbb{F}_q^{n/2} \text{ sub-permutation of } \boldsymbol{x}\}$$

$$L_1 \bowtie L_2 = \{(\boldsymbol{x}_1, \boldsymbol{x}_2) \,|\, \exists \boldsymbol{z}, (\boldsymbol{x}_1, \boldsymbol{z}) \in L_1 \text{ and } (\boldsymbol{x}_2, \boldsymbol{z}) \in L_2\}$$

Time-memory trade-off

Proposition (Complexity)

$$\mathcal{T} = \mathcal{O}\left(|L_1| + |L_2| + |L_1 \bowtie L_2|\right)$$
$$\mathcal{M} = \mathcal{O}\left(|L_1| + |L_2|\right)$$

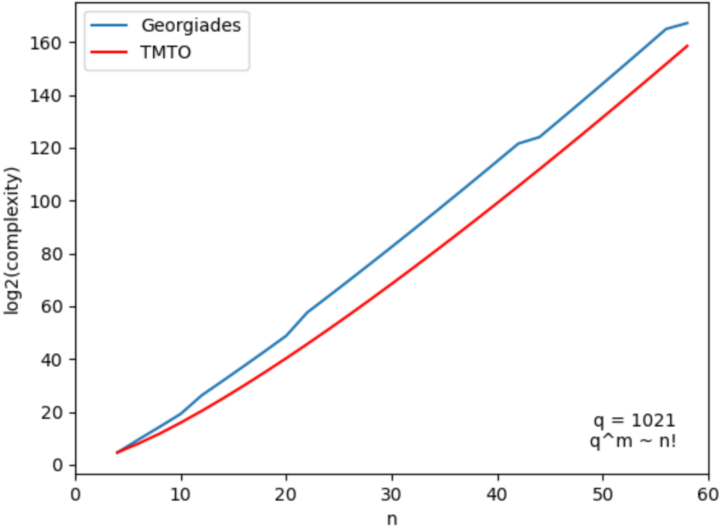
with

$$|L_1| = |L_2| = \frac{n!}{(n/2)!}$$

$$|L_1 \bowtie L_2| = \frac{|L_1| \times |L_2|}{q^m}$$

$\Longrightarrow$ equivalent to Birthday Decoding

Comparison


Georgiades
TMTO
log2(complexity)
n
q = 1021
q^m ~ n!
0
20
40
60
80
100
120
140
160
0
10
20
30
40
50
60

KMP algorithm [KMP19]

Meet in the middle approach between Georgiades and TMTO

$$\begin{pmatrix} \boldsymbol{I}_{m-u} & \multicolumn{2}{c}{\boldsymbol{H}'} \\ \boldsymbol{0} & \boldsymbol{H}_2 & \boldsymbol{H}_3 \end{pmatrix} \begin{pmatrix} \boldsymbol{x}_1 \\ \hline \boldsymbol{x}_2 \\ \hline \boldsymbol{x}_3 \end{pmatrix} = \begin{pmatrix} \boldsymbol{y}_1 \\ \boldsymbol{y}_2 \end{pmatrix}$$

$$\boldsymbol{x}_1 = \boldsymbol{y}_1 - \boldsymbol{H}'(\boldsymbol{x}_2, \boldsymbol{x}_3)$$

$$\boldsymbol{H}_2\boldsymbol{x}_2 + \boldsymbol{H}_3\boldsymbol{x}_3 = \boldsymbol{y}_2$$

KMP algorithm [KMP19]

Proposition (Complexity)

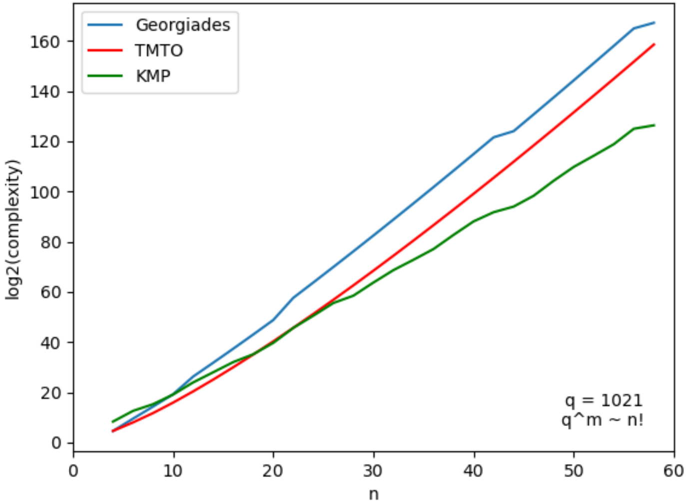
$$\mathcal{T} = \mathcal{O}\left(|L_1| + |L_2| + |L_1 \bowtie L_2|\right)$$

with

$$|L_1| = |L_2| = \binom{n}{(n-m+u)/2}\left((n-m+u)/2\right)!$$

$$|L_1 \bowtie L_2| = \frac{|L_1| \times |L_2|}{q^u}$$

Comparison



*KMP/SBC cost estimation courtesy of `https://github.com/Crypto-TII/CryptographicEstimators`

Other attacks on IPKP

- [BCCG93]
- [PC94]
- Joux-Jaulmes attack [JJ01]

Outline

A variant of the Permuted Kernel Problem

Definition (r-IPKP)

Let $m < n$ and t be positive integers, Given

- $\boldsymbol{H} \in \mathbb{F}_q^{m \times n}$;
- $(\boldsymbol{x}_1, \ldots, \boldsymbol{x}_t) \in (\mathbb{F}_q^n)^t$;
- $(\boldsymbol{y}_1, \ldots, \boldsymbol{y}_t) \in (\mathbb{F}_q^m)^t$,

the Relaxed Inhomogeneous Permuted Kernel Problem r-IPKP$_{q,m,n,t}$ asks to find a permutation $\pi \in \mathcal{S}_n$ such that

$$\boldsymbol{H}\pi\Big[\sum_{i\in[1,t]} \kappa_i \boldsymbol{x}_i\Big] = \sum_{i\in[1,t]} \kappa_i \boldsymbol{y}_i$$

for some $(\kappa_1, \ldots, \kappa_t) \in (\mathbb{F}_q)^t \setminus \{(0, \ldots, 0)\}$.

Number of solutions

Proposition

The average number of solutions for a random r-IPKP$_{q,m,n,t}$ *instance is*

$$\frac{n!}{q^m} \cdot \frac{q^t - 1}{q - 1}$$

Idea of our attack

- Take the smallest weight vector $\boldsymbol{x}$ in $\langle \boldsymbol{x}_1, \ldots, \boldsymbol{x}_t \rangle$,

$$\boldsymbol{x} = \sum_{i \in [t]} \kappa_i \cdot \boldsymbol{x}_i$$

  of weight w.
- Define

$$\boldsymbol{y} = \sum_{i \in [t]} \kappa_i \cdot \boldsymbol{y}_i$$

  and solve IPKP instance $\boldsymbol{H}\pi[\boldsymbol{x}] = \boldsymbol{y}$
- Adapt KMP algorithm to take advantage of the $n - w$ zeros in $\boldsymbol{x}$.

KMP adaptation with zeros

$$
\begin{pmatrix} \boldsymbol{x}_1 \\ \hline \boldsymbol{x}_2 \\ \hline \boldsymbol{x}_3 \end{pmatrix}
\begin{array}{l} \leftarrow n-w-z \text{ zeros} \\ \leftarrow z/2 \text{ zeros} \\ \leftarrow z/2 \text{ zeros} \end{array}
$$

$$
\begin{pmatrix} \boldsymbol{I}_{m-u} & & \boldsymbol{H}' & \\ \boldsymbol{0} & \boldsymbol{H}_2 & & \boldsymbol{H}_3 \end{pmatrix} = \begin{pmatrix} \boldsymbol{y}_1 \\ \boldsymbol{y}_2 \end{pmatrix}
$$

$$
\boldsymbol{x}_1 = \boldsymbol{y}_1 - \boldsymbol{H}'(\boldsymbol{x}_2, \boldsymbol{x}_3)
$$

$$
\boldsymbol{H}_2\boldsymbol{x}_2 + \boldsymbol{H}_3\boldsymbol{x}_3 = \boldsymbol{y}_2
$$

Our attack

Proposition (Complexity)

$$\mathcal{T} = \mathcal{O}\left(\mathcal{T}_{ISD} + \left(|L_1| + |L_2| + |L_1 \bowtie L_2|\right) P\right)$$

with

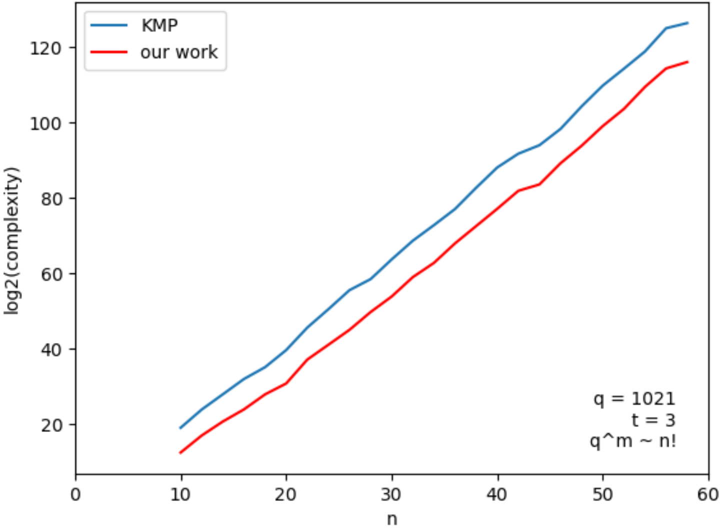
$$k = (n - m + u)/2\,,\; z \leq n - w$$

$$|L_1| = |L_2| = \binom{k}{z/2}\binom{n-z}{k-z/2}(k - z/2)!$$

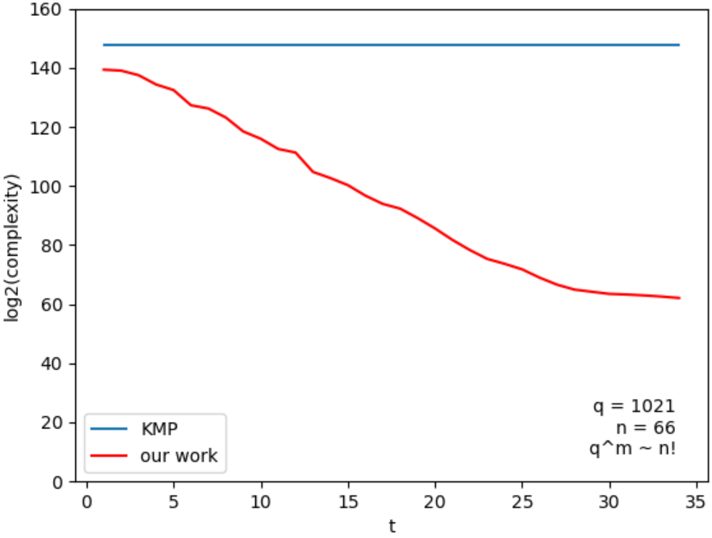
$$|L_1 \bowtie L_2| = \frac{|L_1| \times |L_2|}{q^u}$$

$$P = \frac{\binom{n}{n-w}}{\binom{n-2k}{n-w-z}\binom{k}{z/2}^2}$$

Comparison with KMP



Comparison with KMP for higher t



Outline

Multi-dimensional IPKP

Definition (IPKP)

Let $m < n$ and t be positive integers, Given

- $\boldsymbol{H} \in \mathbb{F}_q^{m \times n}$;
- $(\boldsymbol{x}_1, \ldots, \boldsymbol{x}_t) \in (\mathbb{F}_q^n)^t$;
- $(\boldsymbol{y}_1, \ldots, \boldsymbol{y}_t) \in (\mathbb{F}_q^m)^t$,

the Inhomogeneous Permuted Kernel Problem $\text{IPKP}_{q,m,n,t}$ asks to find a permutation $\pi \in \mathcal{S}_n$ such that

$$\boldsymbol{H}\pi[\boldsymbol{x}_i] = \boldsymbol{y}_i$$

for all $i \in [1, t]$.

Number of solutions

Proposition

The average number of solutions for a random $\mathrm{IPKP}_{q,m,n,t}$ *instance is*

$$\frac{n!}{q^{mt}}$$

$\mathrm{IPKP}_{q,m,n,1}$	$\text{r-IPKP}_{q,m,n,t}$	$\mathrm{IPKP}_{q,m,n,t}$
$\frac{n!}{q^m}$	$\frac{n!}{q^m} \cdot \frac{q^t-1}{q-1}$	$\frac{n!}{q^{mt}}$

Why do we need to consider multi-dimensional IPKP?

Normally with a random instance there is no solution for our parameters.
However, for the signature protocol there exists a permutation π that is a solution to multi-dimensional IPKP.

KMP algorithm, multi-dimensional

Only the size of $L_1 \bowtie L_2$ changes.

Proposition (Complexity)

$$\mathcal{T} = \mathcal{O}\left(|L_1| + |L_2| + |L_1 \bowtie L_2|\right)$$

with

$$|L_1| = |L_2| = \binom{n}{(n-m+u)/2}\left((n-m+u)/2\right)!$$

$$|L_1 \bowtie L_2| = \frac{|L_1| \times |L_2|}{q^{ut}}$$

SBC algorithm [SBC22]

KMP algorithm with ISD.

$$\begin{pmatrix} \boldsymbol{I}_{m-u} & & & \boldsymbol{H}' & \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{H}_2' & & \boldsymbol{H}_3 \end{pmatrix} \begin{pmatrix} \boldsymbol{x}_1 \\ \hline \boldsymbol{x}_2 \\ \hline \boldsymbol{x}_2' \\ \hline \boldsymbol{x}_3 \end{pmatrix} = \begin{pmatrix} \boldsymbol{y}_1 \\ \boldsymbol{y}_2 \end{pmatrix}$$

Link between **homogeneous** PKP and SEP [SBC22]

Definition (Permutation/Subcode Equivalence Problem (PEP/SEP))

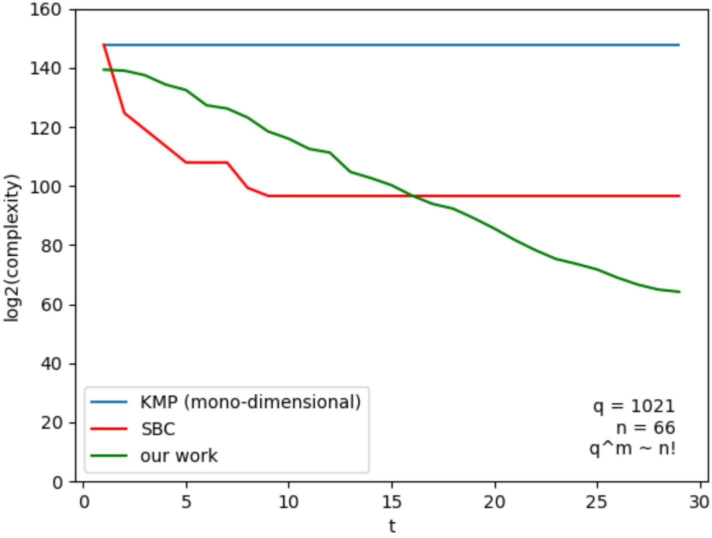
Let $k' \leq k \leq n$. Given two codes $\mathcal{C}[n, k]$ and $\mathcal{C}'[n, k']$, does there exists a permutation π such that

$$\pi[\mathcal{C}'] \subseteq \mathcal{C}?$$

IPKP	Equivalent problem	Parameters
$t < n - m$	SEP	$k = n - m, k' = t$
$t = n - m$	PEP	$k = k' = n - m$
$t > n - m$	SEP	$k = t, k' = n - m$

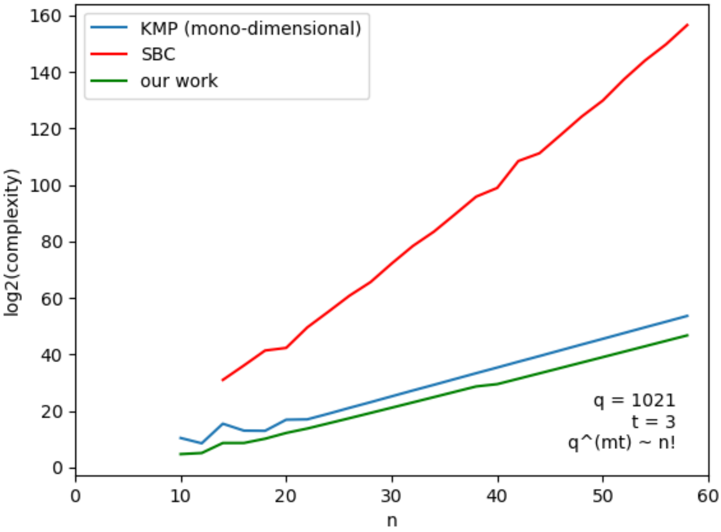
Table: Relations between PKP, SEP and PEP, and corresponding parameters

Comparison

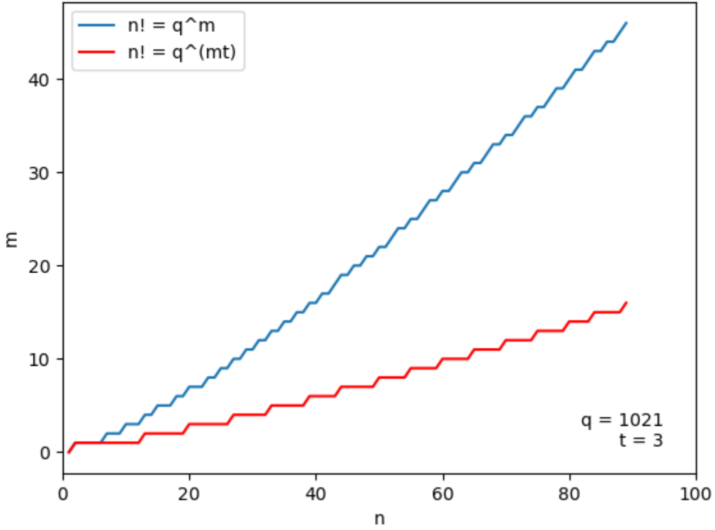

160
140
120
100
80
60
40
20
0
log2(complexity)
0
5
10
15
20
25
30
t
KMP (mono-dimensional)
SBC
our work
q = 1021
n = 66
q^m ~ n!

Outline

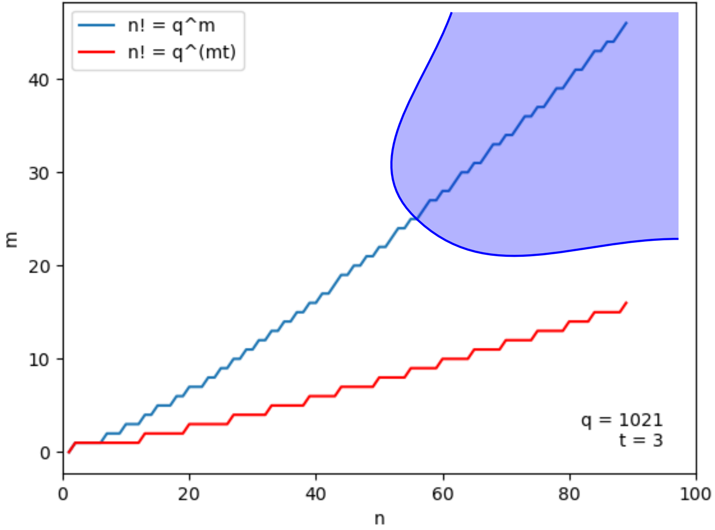
What happens with a different density?



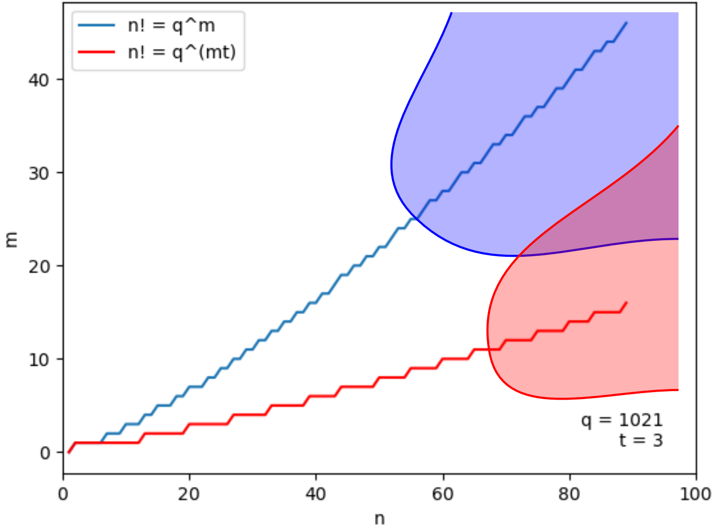
Mitigating both attacks



Mitigating both attacks



Mitigating both attacks



Conclusion

PERK was submitted to the NIST on-ramp call for digital signatures with the following augmented team:

Najwa Aaraj, Technology Innovation Institute, UAE

Slim Bettaieb, Technology Innovation Institute, UAE

Loïc Bidoux, Technology Innovation Institute, UAE

Alessandro Budroni, Technology Innovation Institute, UAE

Victor Dyseryn, XLIM, University of Limoges, France

Andre Esser, Technology Innovation Institute, UAE

Philippe Gaborit, XLIM, University of Limoges, France

Mukul Kulkarni, Technology Innovation Institute, UAE

Victor Mateu, Technology Innovation Institute, UAE

Marco Palumbi, Technology Innovation Institute, UAE

Lucas Perin, Technology Innovation Institute, UAE

Jean-Pierre Tillich, INRIA, Paris, France

Perspectives

- Combinatorial attacks
  - Refine our attack
  - Exploit the multiple instances directly in KMP?
- Algebraic attacks
  - Modelling of permutations in a PhD thesis [Sae17]
  - Polynomial attack when mt is sufficiently high (ongoing work)
  - No efficient attack derived so far in the typical regime
  - Work in progress

Thank you for your attention !

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